THE ACCURACY OF THRUST IN FLIGHT DERIVED FROM ENGINE CALIBRATIONS IN AN ALTITUDE TEST FACILITY

J. C. Ascough
National Gas Turbine Establishment
Pyestock, Farnborough, Hampshire, England

ABSTRACT

Thrust in flight is derived from readings of engine parameters which are preferably calibrated in an Altitude Test Facility. A new refined theory is proposed for estimation of engine calibration uncertainty and its transfer to the in-flight thrust calculations of a multi-engined aircraft. The older simpler theory is shown to over-estimate the in-flight uncertainties. Examples are given of various possible arrangements for the engine calibrations using both an Altitude Test Facility, and a Sea Level Static Test Bed and of the application of these calibrations to flight measurements.

NOTATION

Symbol	Description	Units
A	either (a) flow area or (b) $IC(F_N:C_G)$	m^2 non-D
ATF	Altitude Test Facility	
B	either (a) "2σ" limit of Bias or (b) $IC(F_N:C_{D8})$	as approp. non-D
C_D	Discharge Coefficient	non-D
C_G	Thrust Coefficient by "AP" method	non-D
C_X	Thrust Coefficient by "$W\sqrt{T}$" method	non-D
EL( )	"2σ" Error Limit of ( )	as ( )
F_G	Gross Thrust	kN
F_N	Net Thrust	kN
$IC(y:x_i)$	Influence Coefficient ie % ch in y for 1% ch in x_i	as y/x_i
LCV	Lower Calorific Value of fuel	J/kg
ℓ	number of engines	non-D
m	number of tests in Class II	non-D
n	number of items in Class I	non-D
P	probability	non-D
P_{cell}	cell static pressure	kPa
P_s	static pressure	kPa
P_t	total pressure	kPa
q	Kinematic pressure ($\frac{1}{2}\rho V^2$)	kPa
S	either (a) estimate of Standard Deviation or (b) surface area	as approp. m^2
SLSTB	Sea Level Static Test Bed	
T_s	static temperature	K
T_t	total temperature	K
t_{95}	95% value of Student's "t"	non-D
U	"2σ" limit of Uncertainty	as approp.
W	mass flow rate	kg/s
x_i	general input parameters	as approp.
y	general output result	as approp.
σ( )	Standard Deviation of ( )	as ( )

Suffices

0	free stream (upstream infinity)
1	intake lip
2	compressor face
3	combustion chamber inlet
4	turbine inlet
5	turbine exit
6	reheat inlet
7	nozzle inlet
8	nozzle throat
9	nozzle exit
00	downstream infinity
A	air meter
a	one example of a parameter
b	another example of a parameter
FC	combustion chamber fuel
FR	reheat fuel
i	general input parameter

1. INTRODUCTION

The 'measurement' of the drag of an aircraft in flight is a very indirect process - there is no drag balance available in the sky as there is in the wind tunnel. Hence the drag is inferred as being equal to the thrust, with allowances for accelerations and changes of altitude. In practice a detailed bookkeeping system must be agreed between airframe and engine interests so that the interface between thrust and drag is properly accounted. Items such as intake spillage drag and exhaust jet interference drag must be mutually agreed.

Having reached this agreement on the thrust/drag interface, the problem simplifies somewhat to the in-flight 'measurement' of standard net thrust. Even this is indirect. The gross thrust can only be found from correlations against various parameters such as pressures, temperatures and areas which can be measured in flight. The same applies to the so-called 'ram drag' for which purpose the mass flow is also found from correlations against suitable measurable parameters in flight.

These correlations for gross thrust and mass flow must be established by engine calibrations, preferably in an Altitude Test Facility (ATF), or possibly in a Sea Level Static Test Bed (SLSTB). The uncertainty of the engine calibration coefficients is transferred to the aircraft in flight. This uncertainty transfer process involves the concepts of different error classes, of linked thrust methodology, and of independence between engines. The treatment has been over-simplified in the past leading to over-estimation of uncertainty of the in-flight thrust and to improper decisions about the relative values of ATF vs SLSTB engine calibrations.

The main purpose of the present Paper is to explain the more refined theory of uncertainty-transfer from engine calibration to the aircraft in flight. The paper concentrates on a single thrust method, ie a linked "AP" method using calibrated nozzle coefficients, in order to explain the uncertainty-estimation procedures. But in practice it is recommended that many different in-flight thrust options be kept open and that the final flight test results should take cognizance of results from several options, possibly by the use of a weighted mean value.

2. PRINCIPLES OF ERROR ESTIMATION

2.1 Definition of terms

Attempts have been made in recent years to define the words "Accuracy" and "Precision". For example Abernethy[1], and also a British Standard[2] both propose that "Accuracy" be associated with absence of "Bias" or "Systematic error", while "Precision" be associated with absence of "Random Error". The present writer is sympathetic to these definitions, although it is difficult to apply the words consistently because they are already in such common use in many other senses. A new difficulty has recently become apparent which is that some errors can be regarded as systematic in the short term, but random in the long term. To overcome this difficulty such error has been assigned to a numerical Class II (see Section 2.3.1) and the words "Accuracy" and "Precision" have been freed from control.

A logical difficulty with words like "Accuracy" and "Precision" is that their emotive sense is opposite to their numerical sense. When errors increase the "Accuracy" increases numerically, from, say ±1 per cent to ±2 per cent - but emotively this is a decrease in accuracy! This difficulty can be avoided by the use of the word "Uncertainty".

Abernethy[1] defines "Uncertainty", U as the sum of a Bias and a Precision.

$$U = \pm(B + t_{95}\,S) \qquad (1)$$

The present text uses the word "Uncertainty" to indicate a "2σ" Error Limit, EL (see Appendix A1) but instead of the simple arithmetic addition of Bias, B and Precision, $t_{95}S$ as in Equation (1), it is thought to be more logical to regard B as the semi-range of a rectangular distribution and then to combine by root-sum-squares, RSS because B and S are independent of each other (see Appendix A3):-

$$U = \sqrt{B^2 + [t_{95}S]^2} \qquad (2)$$

The formal justification of this approach is given by Dietrich[3]. However, Equation (2) must be treated with great care as Bias, B is in a different error class (Class III) from Precision, $t_{95}S$ (Class I) as explained below. If these distinctions are not noted, then it might be safer to stick to Abernethy's Equation (1) for general use. But the class-qualified RSS treatment is required for the refined theory of the present text.

It is important to understand and recognise the difference between an individual "Error" and an "Error Limit" (see Appendix A1). A difficulty is that an individual "Error" can not usually be seen; nevertheless it is an important theoretical concept as a single, particular value of discrepancy. The "Error Limits" are the range of values between which the individual "Error" probably lies.

It is worth pointing out the two possible opposite directions of approach which can be employed in Error Estimation. The forward direction is called "Prediction Synthesis" in which the uncertainty of a result, which may not have occurred yet, is predicted by considering the combination of contributing sources of error. The backward direction is called "Post Test Analysis" in which actual experimental results are examined in order to deduce the uncertainty of the true value. The examples in the following Sections of this Paper concern only "Prediction Synthesis".

2.2 The Question of Independence in the Combination of Errors

The central theme running through this Paper is the need to consider whether errors are Independent or Common before attempting to calculate their combined Uncertainty. A Root-Sum-Squares combination, shown in its simplest form in Appendix A2, is justified only if the two items x_a and x_b are independent. If any error is common to both x_a and x_b, ie linked by some definite effect, then a RSS combination is not justified and an arithmetic combination should be applied instead, as explained in Appendix A3.

Unfortunately, both Independent and Common errors are mixed together in most real life situations and so the valid treatment becomes very complicated. Section 3 of this Paper attempts to unravel one such complicated case.

The Question of Independence has many aspects as discussed in Section 2.3 below.

2.3 Aspects of error independence

2.3.1 Classes of error

The basis of this classification is the degree of activity of the error during certain time scales. Those errors which change quickly between one scan and the next are assigned to Class I. This has been associated with the word "Precision". Class I errors are completely Independent time-wise, and hence Error Limits of successive readings may be combined by RSS to get the EL of the mean value. At the other extreme, those errors which remain fixed over a long time scale, covering a complete test series, are assigned to Class III. This has been associated with Systematic Error or Bias. Class III errors are completely Common, time-wise. Now, there is often reason to suspect the existence of an intermediate type of error which remains constant (ie Common, time-wise) during the course of one test period, but which shifts to some other level (ie Independent time-wise) for the next test period. An example might be room temperature affecting an experiment on one day, changing to another level on the next day. This is assigned to Class II. Thus we have:

- Class I: short term random error during a single test period
- Class II: medium term random error between different tests, but fixed during one test
- Class III: long term systematic error.

The main reason for using these error classes is to prevent a false impression of 'accuracy' in the mean value of n test points taken during a single test period - each of these test points would have a fixed Class II error, and a fixed Class III error, which are not reduced by taking the mean value (ie the RSS process is invalid between Common items). To formulate a rule, suppose there are n test points in each of m different tests and an overall mean value is found of the result:

$$\text{Overall mean value, } \bar{\bar{y}} = \frac{\Sigma y}{m.n} \tag{3}$$

Then the Error Limit of $\bar{\bar{y}}$ is:

$$EL_{I,II,II}(\bar{\bar{y}}) = \sqrt{\frac{1}{mn}\left[EL_{I}(y)\right]^2 + \frac{1}{m}\left[EL_{II}(y)\right]^2 + \left[EL_{III}(y)\right]^2} \tag{4}$$

where $EL_I(y)$ is the Class I "2σ" Error Limit

$EL_{II}(y)$ is the Class II "2σ" Error Limit

$EL_{III}(y)$ is the Class III "2σ" Error Limit

2.3.2 Independent and common errors between engines

It has been noted experimentally that the uncertainty of the total thrust of a multi-engined aircraft, expressed as a percentage, is less than that of a single engine. Reference 4 illustrates this phenomenon for the 6-engined XB-70 airplane.

In general, for an aircraft with ℓ engines, suppose:

$$EL(\text{one engine } F_N) = 10 \text{ per cent, say}$$

$$= \frac{1}{\ell} \times 10\% \text{ of total } F_N \tag{5}$$

If the engines are Independent then:

$$EL(\text{total } F_N \text{ from } \ell \text{ engines}) = \sqrt{\sum_1^{\ell}\left[\frac{1}{\ell} \times 10\right]^2}\ \%$$

$$= \frac{1}{\sqrt{\ell}} \times 10\% \tag{6}$$

If each engine had different numerical values of independent EL, we would have:

$$EL(\text{total } F_N \text{ from } \ell \text{ engines}) = \sqrt{\sum_1^{\ell}\left[\frac{EL(\text{each engine } F_N)}{\ell}\right]^2} \tag{7}$$

If the errors were Common to all engines then:

$$EL(\text{total } F_N \text{ from } \ell \text{ engines}) = \ell \times \left[\frac{1}{\ell} \times 10\%\right]$$

$$= 10\% \tag{8}$$

ie just the same as for one engine.

In practice some errors are Independent eg nozzle areas, while other errors are Common eg calorific value of fuel. In such cases of mixed Independent and Common errors, the Error Limit of F_N for the multi-engined aircraft would be:-

(a) <u>with similar numerical value independent EL in each engine</u>
(ie all engines calibrated in same test facility)

$$\%EL(\text{total } F_N \text{ from } \ell \text{ engines}) = \left\{\left[\frac{1}{\sqrt{\ell}} \times \%EL(\text{each engine } F_N \text{ due to indepen errors})\right]^2 + \left[\%EL(\text{each engine } F_N \text{ due to common errors})\right]^2\right\}^{\frac{1}{2}} \tag{9}$$

(b) <u>with different numerical value independent EL in each engine</u>
(ie engines calibrated in different test facilities)

$$\%EL(\text{total } F_N \text{ from } \ell \text{ engines}) = \left\{\sum_1^{\ell}\left[\frac{\%EL(\text{each engine } F_N \text{ due to indepen errors})}{\ell}\right]^2 + \left[\%EL(\text{each engine } F_N \text{ due to common errors})\right]^2\right\}^{\frac{1}{2}} \tag{10}$$

2.3.3 Linked thrust methodology

In the preceding Section a situation was described in which independent errors lead to a more beneficial result than common errors. In this Section the opposite result obtains: common errors are more beneficial. The essential difference is that here the quantities with Common error (ie Gross Thrust and 'Ram Drag') are to be subtracted: previously the quantities (individual engine Net Thrusts) were summed.

The phenomenon has been well explained by Burcham[5] who used the title "TTW method" for the case of common errors and "AP method" for independent errors. The present writer prefers the title "Linked Methodology" for the case of common errors.

A brief reminder of the explanation is as follows. Net thrust is given by:

$$F_N = F_{G9} - F_{G0} \tag{11}$$

If the errors in F_{G9} and F_{G0} are Independent then from Equation (A14) of the Appendix

$$\frac{EL(F_N)}{F_N} = \sqrt{\left[IC(F_N:F_{G9}) \times \frac{EL(F_{G9})}{F_{G9}}\right]^2 + \left[IC(F_N:F_{G0}) \times \frac{EL(F_{G0})}{F_{G0}}\right]^2} \tag{12}$$

To illustrate by numerical example, suppose we had an F_G/F_N ratio of 2, say, then:

$$IC(F_N:F_{G9}) = 2$$

and

$$IC(F_N:F_{G0}) = -1$$

and putting the Independent ELs of F_{G9} and F_{G0} equal to one per cent each, then:

$$\frac{EL(F_N)}{F_N} = \sqrt{\left[2 \times 1\right]^2 + \left[-1 \times 1\right]^2} = 2.24\% \tag{13}$$

If, on the other hand, the errors in F_{G9} and F_{G_0} are Common, then:

$$\frac{EL(F_N)}{F_N} = IC(F_N:F_{G9}) \times \frac{EL(F_{G9})}{F_{G9}} + IC(F_N:F_{G0}) \times \frac{EL(F_{G0})}{F_{G0}} \quad (14)$$

Inserting the same numerical values as before:

$$\frac{EL(F_N)}{F_N} = 2 \times 1 - 1 \times 1 = 1\% \quad (15)$$

which is a big reduction from the 2.24% given by Independent errors.

Any method which leads to Common errors in F_{G9} and F_{G0} will produce this beneficial effect. For example, to use a "W $\sqrt{T}$" correlation for Gross Thrust (ie Burcham's "TTW" method) will automatically involve Common error in F_{G0} due to mass flow. It is also possible for an "AP" method to be "linked" if both F_G and W_8 are correlated against nozzle "AP".

However, if nozzle "AP" is used to give F_{G9}, but some other part of the engine, such as fan correlation is used to give mass flow (ie what Burcham calls his "AP" method), then this is unlinked methodology.

An interesting, and previously unnoticed, aspect of Linked Methodology is to be found in the transfer of calibration error from Engine Test Bed to flight. This is explained in Section 3.2.

2.4 Transfer of a calibration uncertainty (Example of Test Bed Airmeter)

Before dealing with the more complex case of Engine Calibration coefficients in Section 3.0, it is helpful to consider the simpler example of the Test Bed Airmeter calibration.

In a typical installation, when a new airmeter is introduced, it is usual to calibrate it. Special instrumentation rakes are fitted a little downstream of the airmeter measuring plane. Such rakes may carry up to 100 pitot tubes to explore the P_t profile. A static pressure survey, and possibly a temperature survey are also made. Allowances are made for aerodynamic interference between this extra instrumentation and the permanent airmeter instrumentation.

The airmeter is operated over the complete range of mass flow rates, at various levels of temperature and pressure, and readings are taken of the "rake" instrumentations at the same time as readings of the fixed airmeter instrumentation. The results are expressed in the form of values of Airmeter Discharge Coefficient, C_{DA}:

$$C_{DA} = \frac{(W_R \text{ given by rake instrumentation})}{(W_A \text{ given by fixed instrumentation})} \quad (16)$$

A graph of all the results might be as shown in Figure 1 below:

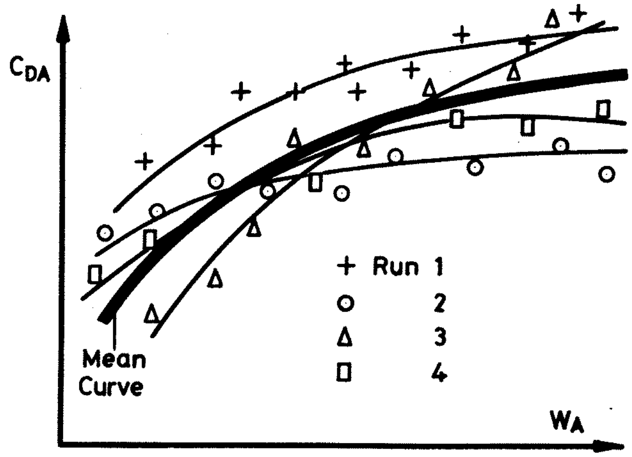

FIG. 1 ILLUSTRATION OF AIRMETER CALIBRATION

In any one run a best curve may be drawn with short term random scatter (Class I) about it. By the very act of fitting the curve the Class I Error Limit is reduced approximately by the factor $1/\sqrt{n}$, like a mean value:

$$EL_I(C_{DA} \text{ curve for single run}) \approx \frac{1}{\sqrt{n}} EL_I \text{ (spot point } C_{DA}) \quad (17)$$

Differences between curves for different runs (Class II) will usually be seen, as shown exaggerated in Figure 1. Suppose there were m different runs, the Error Limit of the overall best curve is reduced approximately by the factor $1/\sqrt{m}$:

$$EL_{II} \text{ (Overall best } C_{DA} \text{ curve)} \approx \frac{1}{\sqrt{m}} EL_{II} \text{ (different } C_{DA} \text{ runs)} \quad (18)$$

In addition there must be long term (Class III) systematic error applied to all results, which is not reduced at all by curve fitting.

Taking all Classes of Error into account, the uncertainty of the overall best curve is:

$$EL_{I,II,III} \text{ (Overall best } C_{DA} \text{ curve)} = \sqrt{\frac{1}{mn}\left[EL_I \text{ (spot point } C_{DA})\right]^2 + \frac{1}{m}\left[EL_{II}(\text{different } C_{DA} \text{ tests})\right]^2 + \left[EL_{III}(C_{DA})\right]^2} \quad (19)$$

The overall best C_{DA} curve is transferred to the test bed computer program for use in all future engine tests. The uncertainty given by Equation (19) is also transferred, but the important thing to note is that all this C_{DA} uncertainty is frozen exclusively into Class III by the act of C_{DA} transfer. It becomes "fossilised". What was once living is now dead and possibly buried out of sight. A moment's thought will convince the reader that Class I scatter, for example, is not transferred as scatter, but is transferred as a contribution to the fixed long term uncertainty of the C_{DA} curve position.

This transfer of airmeter calibration uncertainty is indicated at the top of Table I. The more complicated case of the uncertainties of Engine Test Calibration coefficients C_G, C_{D8}, C_X is described in Section 3.0.

2.5 The use of different thrust measurement methods

A number of distinctly different methods should always be provided for in-flight thrust measurement. Everyone accepts the wisdom of planning a certain amount of redundancy so that if a measurement vital to one method were to fail, then another option could take its place.

There is a tendency for any one organisation to prefer its own method. For example, Reference 6 speaks of the "Lockheed method" (which happens to be "linked AP" in our terminology) and of the "P and W method" (which happens to be "unlinked AP". However, competition is useful in that the variety it stimulates will help to reveal the extent of hidden systematic errors.

During the earlier stages of a new project a large number (20 say) of different combinations of possible methods should be considered. This number would be reduced to manageable size (10 say) by eliminating the least attractive methods with the aid of a Sensitivity Survey. Table 2.1 illustrates the principle by comparing Option 1 against Option 5, but in practice all the possible options should be shown on the same table. Option 1 uses "AP" for F_G, but Option 5 uses "W $\sqrt{T}$", while both obtain mass flow from fan chics. There is no attempt at this stage to introduce Error Classes into the table, but the "Instrumentation" is separated from the "Calibrated Coefficients".

If one of these options had to be thrown out, the axe would fall on Option 1 with its EL(F_N) = 4.2% due to instrumentation, compared with 2.3% for Option 5.

Another use of the Sensitivity Survey table is to direct early attention to the critical items of measurement. In the case of Option 1 the most critical item is nozzle inlet pressure P_{S7} - a modest 2% EL in P_{S7} produces 3.6% EL in F_N due to the large influence coefficient of 1.8. In the case of Option 5 the most critical item is reheat fuel flow - the influence coefficient is only 0.5, but the large EL in W_{FR} of 4% produces 2% EL in F_N. Thus effort can be directed to improve these critical items of instrumentation at an early stage of a new project.

However, there is more to selection of method than the features examined in a sensitivity survey. The validity of the various methods has also to be considered, ie does the calibration which has been derived in the closely controlled conditions of an engine test bed actually apply to the flight situation? But it is not possible to discuss this further in the present Paper.

Therefore, when flight testing begins, a modest number of options should remain available for use.

TABLE 2.1 Example of simple Sensitivity Survey (Single engined fighter aircraft)

Flight condition: supersonic cruise with reheat on
Type of output: $y = F_N$
$\frac{F_G}{F_N}$ ratio = 1.6

Input parameter x_i		Error Limit EL (x_i)	Option 1 "AP" method IC(y:x_i)	Option 1 "AP" method EL × IC %	Option 5 "W $\sqrt{T}$" method IC(y:x_i)	Option 5 "W $\sqrt{T}$" method EL × IC %
Calibrations etc						
Full scale nozzle C_X carpet		1.5%	-	-	1.6	2.4
Full scale nozzle C_G carpet		1.5%	1.6	2.4	-	-
Full scale nozzle C_{D8} carpet		1.5%	-	-	-	-
Fan chic		1.5%	-0.6	-0.9	-0.6	-0.9
Fuel cal. val. LCV		1.0%	0	0	0.5	0.5
% EL (y) = $\sqrt{\Sigma [\%EL \times IC]^2}$ →			-	2.6%	-	2.6%
Instrumentation						
Eng. face	P_{t2}	1.0%	-0.6	-0.6	0.3	0.3
Eng. face	T_{t2}	1.0%	0.4	0.4	-0.2	-0.2
Free stream	P_{SO}	0.5%	0.2	0.1	0.2	0.1
By-pass duct	ΔP_{13}	1.0%	-	-	-	-
By-pass duct	P_{S13}	1.0%	0.9	0.9	-0.5	0.5
Noz. inlet	P_{S7}	2.0%	1.8	3.6	0.4	0.8
CC fuel flow	W_{FC}	2.0%	-	-	0.1	0.2
RH fuel flow	W_{FR}	4.0%	-	-	0.5	2.0
LP spool	N_L	0.5%	-1.1	-0.55	0.5	0.25
Noz. area	A_8	2.0%	0.9	1.8	0.1	0.1
Power offtake	Q	0.5%	-	-	0	0
Services bleed	W_B	1.0%	-	-	-0.4	-0.4
% EL (y) = $\sqrt{\Sigma [\%EL \times IC]^2}$ →			-	4.2%	-	2.3%

The problem is how to derive a single figure for thrust which goes forward to the drag analysis. Careful comparison of different options often leads to elimination of several of them due to malfunctioning instrumentation or obvious invalidity of the method. In the end, however, a few different answers will remain which the engineer will have considered both free of instrumentation error and of equal validity. Traditionally a single preferred method is selected (perhaps with the aid of the Sensitivity Survey Table) and the results from this one method only are published. Often, however, there is controversy over the rival result of an alternative method. This situation can be avoided by the use of the Weighted Mean Value, which combines the results of all the different options in the most efficient way.

Suppose y_r is the result of the rth option which has the Error Limit, $EL(y_r)$, then the statistical weight of that result is:

$$w_r = \frac{1}{\left[EL\ (y_r)\right]^2} \qquad (20)$$

The Weighted Mean Value of n different results is

$$y_{WM} = \sum_{r=1}^{n} w_r\, y_r \Big/ \sum_{r=1}^{n} w_r \qquad (21)$$

where the Weight of the Weighted Mean is

$$w_{WM} = \sum_{r=1}^{n} w_r \qquad (22)$$

and the Error Limit of the Weighted Mean is:

$$EL\ (y_{WM}) = \sqrt{\frac{1}{w_{WM}}} \qquad (23)$$

Figure 2 illustrates an example where the results are Drag Coefficients given by 3 different options of thrust measurement in flight. In example (i) the 3 C_D results of 0.050, 0.055 and 0.060 have similar Uncertainties or Error Limits of ±9 per cent, ±10 per cent and ±11 per cent. The Weighted Mean Drag Coefficient from Equation (21) is:

$$C_{D,WM} = \frac{\frac{1}{9^2} \times 0.050 + \frac{1}{10^2} \times 0.055 + \frac{1}{11^2} \times 0.060}{\frac{1}{9^2} + \frac{1}{10^2} + \frac{1}{11^2}}$$

$$= 0.0544 \qquad (24)$$

and the Uncertainty from Equation (23) is:

$$EL\ (C_{D,WM}) = \sqrt{\frac{1}{\frac{1}{9^2} + \frac{1}{10^2} + \frac{1}{11^2}}} = 5.7\% \qquad (25)$$

Thus, in this example, as illustrated in Figure 2 (i) the Uncertainty of the Weighted Mean is much less than that of any of the single options and this is always the case when separate options have similar Uncertainty.

In example (ii), the 3 C_D results are the same as in example (i) viz: 0.050, 0.055 and 0.060, but this time Option 1 is supposed to have a much smaller Uncertainty of ±5 per cent, than the other two, ±20 per cent and ±25 per cent. The Weighted Mean Drag Coefficient from Equation (21) is:

$$C_{D,WM} = \frac{\frac{1}{5^2} \times 0.050 + \frac{1}{20^2} \times 0.055 + \frac{1}{25^2} \times 0.060}{\frac{1}{5^2} + \frac{1}{20^2} + \frac{1}{25^2}}$$

$$= 0.0506 \qquad (26)$$

and its Uncertainty from Equation (23) is:

$$EL\ (C_{D,WM}) = \sqrt{\frac{1}{\frac{1}{5^2} + \frac{1}{20^2} + \frac{1}{25^2}}} = 4.8\% \qquad (27)$$

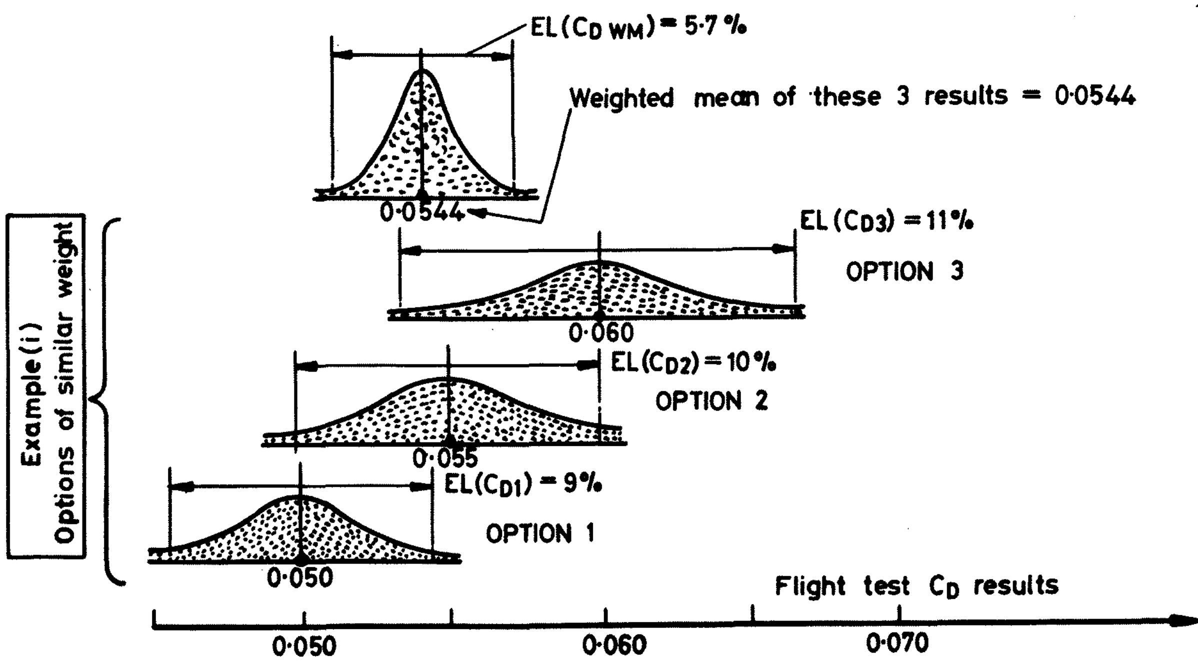

FIG. 2 (i) WEIGHTED MEAN FLIGHT TEST RESULT

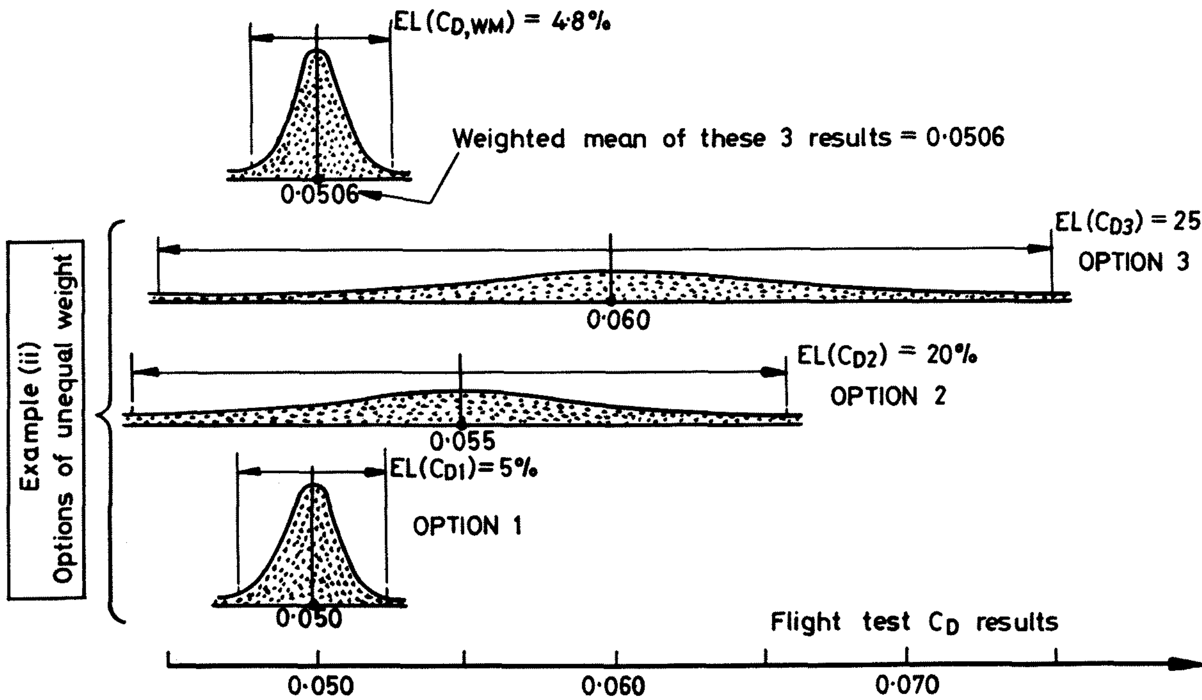

FIG. 2 (ii) WEIGHTED MEAN FLIGHT TEST RESULT

Thus, in this example, as illustrated in Figure 2 (ii) the Uncertainty is only a little better than that of the best option (Option 1), while the Weighted Mean Drag Coefficient is very close to that of the best option.

The lesson from example (ii) is that if any one option clearly has a much smaller Uncertainty than the rival options, then this one good option can be accepted straight away as the definitive result (although it would do no harm to calculate the weighted mean). But the situation is more likely to be as example (i) in which no single option is clearly the best. In this general case the Weighted Mean will produce a valuable reduction in the Uncertainty of the Drag Coefficient from the flight tests.

3. REFINED THEORY OF UNCERTAINTY PREDICTION FOR IN-FLIGHT THRUST MEASUREMENT OF A TWIN-ENGINED AIRCRAFT

3.1 Engine test calibration uncertainty

The object of an engine calibration in a test facility is to establish correlation curves between instrumentation readings (which readings can also be taken in flight) and the thrust and mass flow (which can not be measured directly in flight). The most convenient correlations are in the form of the coefficients C_G, C_{D8} and C_X plotted against NPR. Thus:

$$C_G = \left[\frac{F_G}{A_8\ P_{so}}\right]_{\text{Test bed measurement}} \Bigg/ \left[\frac{F_G}{A_8\ P_{so}}\right]_{\text{Ideal}} \quad (28)$$

$$C_{D8} = \left[\frac{W_8\sqrt{T_8}}{A_8\ P_{t8}}\right]_{\text{Test bed measurement}} \Bigg/ \left[\frac{W\sqrt{T}}{A\ P_t}\right]_{\text{Ideal}} \quad (29)$$

$$C_X = \left[\frac{F_G}{W_8\sqrt{T_8}}\right]_{\text{Test bed measurement}} \Bigg/ \left[\frac{F_G}{W\sqrt{T}}\right]_{\text{Ideal}} \quad (30)$$

All the various measurements x_i are shown column-wise in Table I, with the influence coefficients $IC(C_G{:}x_i)$ in the next column. Estimates of Error Limits of each x_i for the 3 Classes I, II, III are multiplied by the respective ICs and inserted separately for each Class. Note that $EL(C_{DA})$ for the airmeter only appears as the fossilised Class III as already explained in Section 2.4. The calorific value of the fuel also only occurs as Class III.

The uncertainty of a spot point measurement of the co-fficients C_G, C_{D8} and C_X are calculated by RSS within each Class, thus:

TABLE 3.1 Single engine calibration spot point uncertainties

	Class I	Class II	Class III
%EL (spot point C_G)	0.41	0.99	1.15
%EL (spot point C_{D8})	0.44	1.01	1.14
%EL (spot point C_X)	0.16	0.25	0.26

However, the uncertainty of a spot point is not transferred to flight. Rather, it is the uncertainties of the coefficient curves which are transferred. Assuming that the curves are drawn through n = 4 test points on each of m = 4 different test runs, then the uncertainties are reduced by the factors $1/\sqrt{n}$ and $1/\sqrt{m}$ to become the values shown in Table 3.2.

TABLE 3.2 Single engine calibration curve position uncertainties

	Class I	Class II	Class III	RSS all classes
%EL (C_G curve)	0.21	0.49	1.15	1.5
%EL (C_{D8} curve)	0.22	0.50	1.14	1.4
%EL (C_X curve)	0.08	0.13	0.26	0.3

Note that Class III uncertainties are not reduced by the curve-fitting process.

The way in which the different Classes are combined depends upon whether "linked methodology" is to be used in flight as discussed in Section 3.2 and also upon whether both engines, or only one engine, are calibrated in same test bed, (as discussed in Section 3.3).

3.2 Linked thrust methodology applied to engine calibration coefficients (single-engined aircraft)

Assuming that mass flow in flight, as well as gross thrust, are both to be found by nozzle coefficients then this is an example of "linked methodology", such that common errors in C_G and C_{D8} (such as nozzle area measurement in the test bed) will be partially cancelled in flight. This benefit would be lost if mass flow were to be derived from some other correlation, say from compressor chics, with gross thrust coming from nozzle coefficients.

Treating C_G and C_{D8} separately to begin with, their uncertainty is transferred to flight according to the following equations:

(i) From C_G

$$\frac{EL\ (F_N)}{F_N} = IC\ (F_N{:}C_G) \times \frac{EL\ (C_G)}{C_G} = A \times \frac{EL\ (C_G)}{C_G} \qquad (31)$$

(ii) From C_{D8}

$$\frac{EL\ (F_N)}{F_N} = IC\ (F_N{:}C_{D8}) \times \frac{EL\ (C_{D8})}{C_{D8}} = B \times \frac{EL\ (C_{D8})}{C_{D8}} \qquad (32)$$

However some of the test bed errors causing EL (C_G) are the same ones that cause EL (C_{D8}) so some partial cancellation or reinforcement is to be expected. C_G and C_{D8} are not independent and so a root-sum-squares combination is not valid.

Let us examine the problem numerically. Typically A = 2, and B = -1.3. Now suppose ALL the error in C_G and C_{D8} is due to an error in nozzle area which affects C_G and C_{D8} equally. If C_G and C_{D8} are both misplaced by this error of up to 1 per cent then the error in calculated net thrust will be up to:

$$(2 \times 1) + (-1.3 \times 1) = 0.7\% \qquad (33)$$

On the other hand, if C_G is misplaced by up to 1% due entirely to load cell error which does not influence C_{D8}, and C_{D8} is similarly misplaced by up to 1% due entirely to fuel flow error which does not influence C_G, the likely error in calculated nett thrust is given by

$$\sqrt{(2 \times 1)^2 + (-1.3 \times 1)^2} = 2.4\% \qquad (34)$$

Considering all the test bed errors, some of them affect both C_G and C_{D8}, some affect C_G but not C_{D8}, while others affect C_{D8} but not C_G. Thus C_G and C_{D8} are partly independent, partly non-independent, and so the root-sum-squares combination is invalid with separate C_G and C_{D8} terms. To get round this problem it is necessary to go right back to the test bed errors and note how they are propagated through the C_G and C_{D8} curves all the way to the flight result F_N. Thus for a single test bed parameter, x_i:

$$\left[\frac{EL\ (F_N)}{F_N}\right]_{flight} = \left[IC\left((A\ C_G + B\ C_{D8}) : x_i\right) \times \frac{EL\ (x_i)}{x_i}\right]_{test\ bed} \qquad (35)$$

The explicit non-independent or common relationship between C_G and C_{D8} is thus taken fully into account in Equation (35) and so the remaining Independent elements of the n different x_i parameters may now be combined by RSS, thus:

$$\left[\frac{EL\ (F_N)}{F_N}\right]_{flight} = \sqrt{\sum_{i=1}^{n}\left[IC\left((A\ C_G + B\ C_{D8}) : x_i\right) \times \frac{EL\ (x_i)}{x_i}\right]^2_{test\ bed}} \qquad (36)$$

A more convenient form of Equation (36) can be shown to be:

$$\frac{EL\ (F_N)}{F_N} = \sqrt{(A^2 + AB)\left[\frac{EL\ (C_G)}{C_G}\right]^2 + (B^2 + AB)\left[\frac{EL\ (C_{D8})}{C_{D8}}\right]^2 - AB\left[\frac{EL\ (C_x)}{C_x}\right]^2} \qquad (37)$$

In this equation the common errors in C_G and C_{D8} are cancelled by the "C_x" term, rather like a covariance in a formal statistical treatment.

Referring back to the calibration curves uncertainties in Table 3.2, near the end of Section 3.1:

EL (C_G curve) = 1.5%
EL (C_{D8} curve) = 1.4% } all Classes combined
EL (C_x curve) = 0.3%

and using the typical values of A = 2 and B = -1.3 to insert in Equation (37)

$$\frac{EL\ (F_N)}{F_N} = \sqrt{1.4 \times [1.5]^2 - 0.91 \times [1.4]^2 + 2.6 \times [0.3]^2} = 1.3\% \qquad (38)$$

If it had been assumed that all the errors in C_G and C_{D8} were Common then we would have:

$$\frac{EL\ (F_N)}{F_N} = (2 \times 1.5) + (-1.3 \times 1.4) = 1.2\% \qquad (39)$$

On the other hand, if it had been assumed that errors in C_G and C_{D8} were completely Independent then we would have:

$$\frac{EL\ (F_N)}{F_N} = \sqrt{2^2 \times [1.5]^2 + (-1.3)^2 \times [1.4]^2} = 3.5\% \qquad (40)$$

This latter figure corresponds to the "old theory" which is still in common use. If error in C_G and C_{D8} were the only ones to consider, then the "old theory" would be extremely misleading. In practice, however, the wrongness is alleviated by the impact of other effects.

3.3 Linked thrust methodology applied to engine calibration coefficients (twin-engined aircraft

In a twin-engined aircraft some errors are Common to both engines and some are Independent. If the uncertainty of net thrust for a single engine were $\frac{EL\ (F_N)}{F_N} = 1\%$, say, then

(i) assuming completely Common errors

$$\frac{EL\ (F_{N1} + F_{N2})}{\text{Total } F_N} = \frac{1}{2} + \frac{1}{2} = 1\% \qquad (41)$$

(ii) assuming Independent errors

$$\frac{EL\ (F_{N2} + F_{N2})}{\text{Total } F_N} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = 0.7\% \qquad (42)$$

The distinction here between Common errors and Independent errors of two engines is not as dramatic as it is for the calibration curves of a single engine as described in Section 3.1.

Some of the errors of F_N in flight are Common to both engines - ambient pressure P_{SO} for example is a Common airframe reading. Other errors are Independent - eg the individual engine fuel flow meters.

In this Section we are concerned with a more subtle distinction between Common and Independent errors which occurs with respect to the calibration curves of two engines calibrated consecutively in the same facility. It can reasonably be expected that the Class III errors of the test bed remain constant during both engine calibrations, so that Class III calibration uncertainty must be considered as Common. By definition Class I and II calibration error must be considered as Independent of the other engine.

Splitting the engine calibration errors thus we put:

(a1) "Independent of other engine" (ie those due to Classes I and II in the ATF)

(a2) "Common to both engines" (ie Class III in the ATF).

Another category (b) applies if only that one engine is calibrated in the ATF (other engine is SLSTB). Category (e) relates to the "old theory" of assumed Independence between C_G and C_{D8}.

The calculations, making use of Equation (37) are shown in Table II with values of influence coefficients for the flight condition of 0.9 MN at low altitude, "high power", dry.

$$A = IC\ (F_N : C_G) = 2.18$$
$$B = IC\ (F_N : C_{D8}) = -1.53$$

Values for %EL (C_G curve), %EL (C_{D8} curve) and %EL (C_x curve) for the 3 classes are taken from the bottom of Table I for use in Table II. The results of the calculations are shown in Table 3.3 below:

TABLE 3.3 Uncertainties of linked calibration curves of C_G, C_{D8} and C_x (one engine of twin-engined aircraft

Case	EL (1 engine spot point F_N)
(a1) "Independent of other engine"	0.44%
(a2) "Common to both engines"	0.88%
(b) "Independent of other engine"	0.98%
(e) "Old theory", Independent	3.38%

Another category (c) applies if only that one engine is calibrated on the SLSTB (other engine in ATF). For this the uncertainty of F_N due to C_G and C_D is arbitrarily put 3 times that of category (b).

TABLE 3.4 Uncertainty prediction of in-flight thrust of twin-engined aircraft (due only to engine calibration)

Calibrations		% EL (Spot point twin engine total F_N)		
Engine 1	Engine 2	"High Power"		"Low Power"
[⟵ same ATF ⟶] (a1), (a2)	(a1), (a2)	$\sqrt{\left[\frac{0.44}{\sqrt{2}}\right]^2 + 0.88^2}$	= 0.9%	1.6%
[ATF (b)	SLSTB (c)]	$\sqrt{\left[\frac{0.98}{2}\right]^2 + \left[\frac{3 \times 0.98}{2}\right]^2 + 0}$	= 1.5%	2.7%
[⟵ same SLSTB ⟶] (d1), (d2)	(d1), (d2)	$\sqrt{\left[\frac{3 \times 0.44}{\sqrt{2}}\right]^2 + \left[3 \times 0.88\right]^2}$	= 2.8%	4.9%
[⟵ different ATFs ⟶] (b)	(b)	$\sqrt{\left[\frac{0.98}{\sqrt{2}}\right]^2 + 0}$	= 0.7%	1.2%
[⟵ different SLTBs ⟶] (c)	(c)	$\sqrt{\left[\frac{3 \times 0.98}{\sqrt{2}}\right]^2 + 0}$	= 2.1%	3.6%
[Same ATF "old theory"] (e)	(e)	$\frac{3.38}{\sqrt{2}}$	= 2.4%	2.3%

Further categories (d1) and (d2) apply if both engines are calibrated on the SLSTB. The uncertainties are put 3 times those of (a1) and (a2).

If the error in flight was due entirely to the uncertainty of C_G, C_{D8} and C_x transferred from the engine test calibrations then the uncertainty of total F_N of the twin-engined aircraft would be as in Table 3.4, calculated with Equation (9) or (10) of Section 2.3.2.

Results from a similar calculation for a "low power" flight condition are entered in a column alongside the "high power" ones for comparison.

From inspection of the above results, due entirely to engine calibration uncertainties it is possible to formulate provisionally the conclusions listed in Section 4.0. These will be checked against the complete prediction in Section 3.4, which also takes account of uncertainties of instrumentation readings in flight.

3.4 Complete prediction of in-flight thrust uncertainty

It is necessary to calculate the uncertainties of each single engine (Part 1 of Table III) before dealing with the total thrust of the twin-engined aircraft (Part 2 of Table III).

The uncertainties of the linked calibration curves of C_G, C_{D8} and C_x transferred from the engine test bed are entered near the top of Table III. Just as the airmeter C_{DA} calibration errors were fossilised into a Class III uncertainty upon transfer for engine testing (see Table I), in a similar way the engine calibration errors are fossilised into a Class III uncertainty upon transfer for flight testing in Table III. The extra complication is that they are separated into the two columns: "Independent of the other engine"
or "Common to both engines"
Five possibilities, (a) through (e), are considered for the calibration of one engine vis-a-vis the other.

The uncertainty estimates due to instrumentation readings are also entered in Part 1 of Table III. In the case of "Aircraft instrumentation", the uncertainties are entirely "Common" to both engines. In the case of "Engine instrumentation", the Class I uncertainties are entered as entirely "Independent", the Class II are split between "Independent" and "Common" while the Class III are put as entirely "Common".

With the particular thrust method employed, the nozzle inlet pressure was found from the wall static readings, P_{S7}. The pressures P_{t5} and P_{t15} were not used and so their influence coefficients are zero with this particular thrust method.

The RSS combinations of the separate classes are shown at the bottom of Part 1, keeping "Independent" apart from "Common".

Part 2 of Table III shows various possible engine calibration arrangements. Where the "Independent" ELs are the same for each engine, the $1/\sqrt{\ell}$ factor can be applied as shown in Section 2.3.2, Equation (9). But with combination "b + c" it is necessary to use Equation (10) to give the "Independent" total for the aircraft. Thus the "Independent" ELs are combined with the "Common" by RSS to give the uncertainties of the twin engine spot point $C_T = (F_{N1} + F_{N2})/qS$.

The results are copied into Table 3.5 below together with similar calculations for a "low power" flight condition at low Mach number, low altitude.

TABLE 3.5 Summary of complete prediction of twin engine in-flight thrust uncertainty

Calibrations		% EL (Twin engine spot point C_T)	
Engine 1	Engine 2	"High Power"	"Low Power"
[← same ATF →] (a1) (a2)	(a1) (a2)	1.6%	2.3%
[ATF (b)	SLSTB (c)]	2.0%	3.2%
[← same SLSTB →] (d1) (d2)	(d1) (d2)	3.1%	5.1%
[← different ATFs →] (b)	(b)	1.5%	2.0%
[← different SLSTBs →] (c)	(c)	2.4%	4.0%
[← same ATF "old theory" →] (e)	(e)	2.7%	2.8%

These uncertainties are somewhat higher than those due to engine calibrations alone (see Table 3.4) but the same conclusions can be drawn as listed in Section 4.0 below.

4. CONCLUDING REMARKS

4.1 Comparison of old theory and new theory

The "old theory" [(e) + (e)] in Table 3.5 seriously overestimates the in-flight thrust uncertainty with two engines calibrated in the ATF, compared with the "new theory" [(a1), (a2) + (a1),(a2)]. This is because the "old theory" wrongly assumes complete independence between C_G and C_{D8} whereas in fact there is a significant "common" element when using "linked methodology".

In mitigation of the "old theory", the additional assumption of complete independence between two engines produces a small underestimation of in-flight thrust uncertainty, and it was hoped that these opposing effects would cancel out. But when realistic numerical values are used the overestimation part of the "old theory" is found to swamp the underestimation part.

4.2 Choice of ground facility for engine calibration

Calibration of just a single engine in an ATF is a significant improvement over the case of no ATF calibration, should it be impossible to calibrate both engines in the ATF (compare (b + c) against (d + d)).

A calibration in different facilities of the same type ie 2 ATFs or 2 SLSTBs gives a marginal improvement in accuracy.

4.3 Effect of engine setting

Calculations at high power are significantly more accurate than those at low power (except for the anomalous "old theory").

Acknowledgement

The central feature of this Paper, viz: the refined theory of the propagation of Uncertainty of engine calibration coefficients from test bed to flight, was proposed by Mr. G. P. Wilson of the British Aircraft Corporation Ltd., and developed jointly in collaboration with the author.

Disclaimer

Any views expressed are those of the author and do not necessarily represent those of the UK Ministry of Defence (PE).

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APPENDIX

BASIC PRINCIPLES OF ERROR ESTIMATION

A1. Probability Distributions, errors and error limits

Different measured values of a parameter x, such as a pressure reading, will usually exhibit some variation of errors on either side of a mean value $\bar{x}$. The probability that x will occur within an interval A to B is shown by the hatched area beneath the curve of the probability distribution in Figure A.1.

the shortened notation for which is ±EL(x). Procedures will be described for finding the Error Limits of a result y, EL(y), where y is a calculated result with many input measurements, x_i.

The Standard Deviation, σ(x) can sometimes be estimated from observed sample readings of x:

$$\sigma(x) = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}} \qquad \text{(A2)}$$

This is more likely to be feasible in the case of Class I errors which are by nature amply observable.

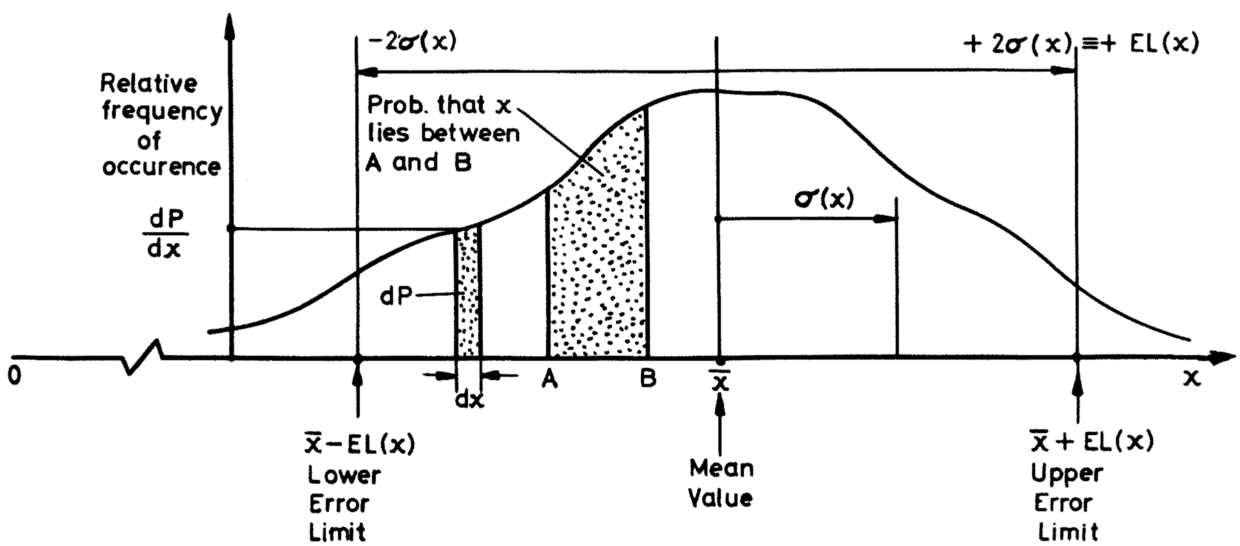

FIG. A1 A PROBABILITY DISTRIBUTION

When the shape of a distribution is known, the standard deviation is given by

$$\sigma(x) = \sqrt{\int_{-\infty}^{+\infty} (x - \bar{x})^2 \frac{dP}{dx} \cdot dx} \qquad \text{(A1)}$$

If the distribution is Gaussian, then about 95 per cent of possible values of x lie within the range ±2σ(x). If the distribution is Rectangular, then 100 per cent of x values lie within ±2σ(x). The present text will deal with such "2σ" Error Limits,

In the case of Class II and Class III errors, these are less and less observable and it becomes necessary to make shrewd guesses, backed by whatever evidence is available, of what the Error Limits might be without the benefit of a σ(x) calculation.

A2. Combination of Error Distributions by Root-Sum-Squares

A simple example of the combination of Error Distributions by Root-Sum-Squares (RSS) is given

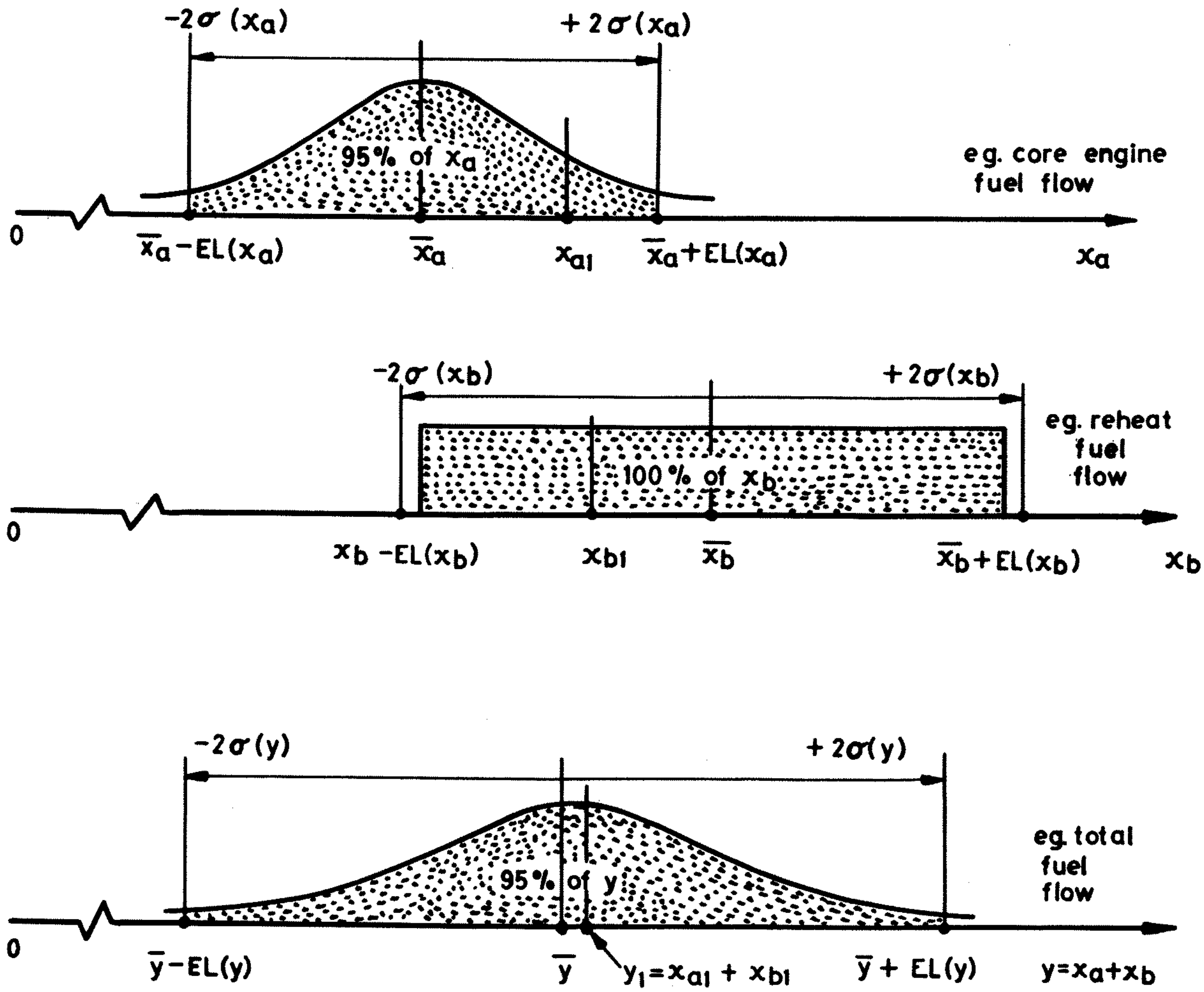

FIG.A2 ROOT SUM SQUARES COMBINATION OF GAUSSIAN AND RECTANGULAR DISTRIBUTIONS

by the sum of a core engine fuel flow x_a having a Gaussian distribution, say, and a reheat fuel flow x_b having a Rectangular distribution, say, as shown in Figure A.2.

In each case the Error Limits of x, ±EL(x), are identified as estimates of the "2σ" bounds, ±2σ(x).

If an individual value of the core engine fuel flow x_a (say x_{a1}) is added to an individual value of the reheat fuel flow x_b (say x_{b1}) the result is a value of total fuel flow:

$$y_1 = x_{a1} + x_{b1} \qquad \text{(A3)}$$

We do not usually know this particular exact result but when all possible results are considered the mean value of total flow is:

$$\bar{y} = \bar{x}_a + \bar{x}_b \qquad \text{(A4)}$$

and the standard deviation of y is:

$$\sigma(y) = \sigma(x_a + x_b) = \sqrt{\sigma^2(x_a) + \sigma^2(x_b)} \qquad \text{(A5)}$$

and the distribution of y will tend towards Gaussian as indicated in the lower part of Figure A2.

A3. Combination of Independent and Common Errors (including use of Influence Coefficients

Considering first the simple example of the sum or difference of two items:

$$y = x_a + x_b \qquad \text{(A6)}$$

$$y = x_a - x_b \qquad \text{(A7)}$$

where for example x_a could be a barometric pressure, x_b a gauge pressure, making y the absolute pressure (either super-atmospheric or sub-atmospheric).

The exact values of x_a and x_b are not known, hence exact y is not known. But the mean value is:

$$\bar{y} = \bar{x}_a + \bar{x}_b \qquad \text{(A8)}$$

$$\text{or} \quad \bar{y} = \bar{x}_a - \bar{x}_b \qquad \text{(A9)}$$

If the errors in x_a and x_b are <u>Independent</u> then the Error Limit of y is:

$$EL(y) = \sqrt{\left[EL(x_a)\right]^2 + \left[EL(x_b)\right]^2} \qquad \text{(A10)}$$

This root-sum-squares combination in Equation (A10) would also apply even if the plus sign (+) of Equation (A6) were replaced by the minus sign (-) of Equation (A7).

If the errors in x_a and x_b are non-Independent or Common i.e. linked to each other by some definite relationship, then the Error Limit of y in Equation (A6) is:

$$EL\ (y) = EL\ (x_a) + EL\ (x_b) \qquad (A11)$$

Supposing now that the plus (+) of Equation (A6) were replaced by the minus (-) of Equation (A7) then the sign of Equation (A11) would also change from (+) to (-):

$$EL\ (y) = EL\ (x_a) - EL\ (x_b) \qquad (A12)$$

These ideas are illustrated in Figure A3.

The important feature to note is that non-Independent or Common errors are self-cancelling when y is the difference of x_a and x_b. This produces a much smaller Error Limit of y than in the case of Independent errors, which is the essential explanation of the advantages of "Linked Methodology" (see Section 2.3.3). Common errors are not always self-cancelling however. For example, in a multi-engine aircraft the Common errors are additive (see Section 2.3.2).

The above theory can be extended to the usually more complicated case where y is a function of several items x_i:

$$y = f\ (x_i) \qquad (A13)$$

Then, providing the errors are completely Independent the Error Limit of y is:

$$\frac{EL\ (y)}{y} = \sqrt{\sum_i \left[IC\ (y:x_i) \times \frac{EL\ (x_i)}{x_i} \right]^2} \qquad (A14)$$

where IC $(y:x_i)$ is the Influence Coefficient of x_i relative to the result y:

$$IC\ (y:x_i) = \frac{dy}{dx_i} \times \frac{x_i}{y} \qquad (A15)$$

An alternative definition of Influence Coefficient is the percentage change in y for a 1 per cent change in x_i, which can be found by running a computer calculation with successive numerical perturbations in the x_i inputs.

If the errors in the various x_i are linked by a definite relationship then instead of Equation (A14) the Error Limit of y is given by

$$\frac{EL\ (y)}{y} = \sum_i \left[IC\ (y:x_i) \times \frac{EL\ (x_i)}{x_i} \right] \qquad (A16)$$

Real situations are complicated by a mixture of both Independent and Common errors. By going back to very basic inputs and also going on to the final output for the Influence Coefficients (e.g. don't stop at $y = F_G$, but go on to $y = F_N$, or even $y = C_{Drag}$), then it should be valid to apply the root-sum-square combination of Equation (A14).

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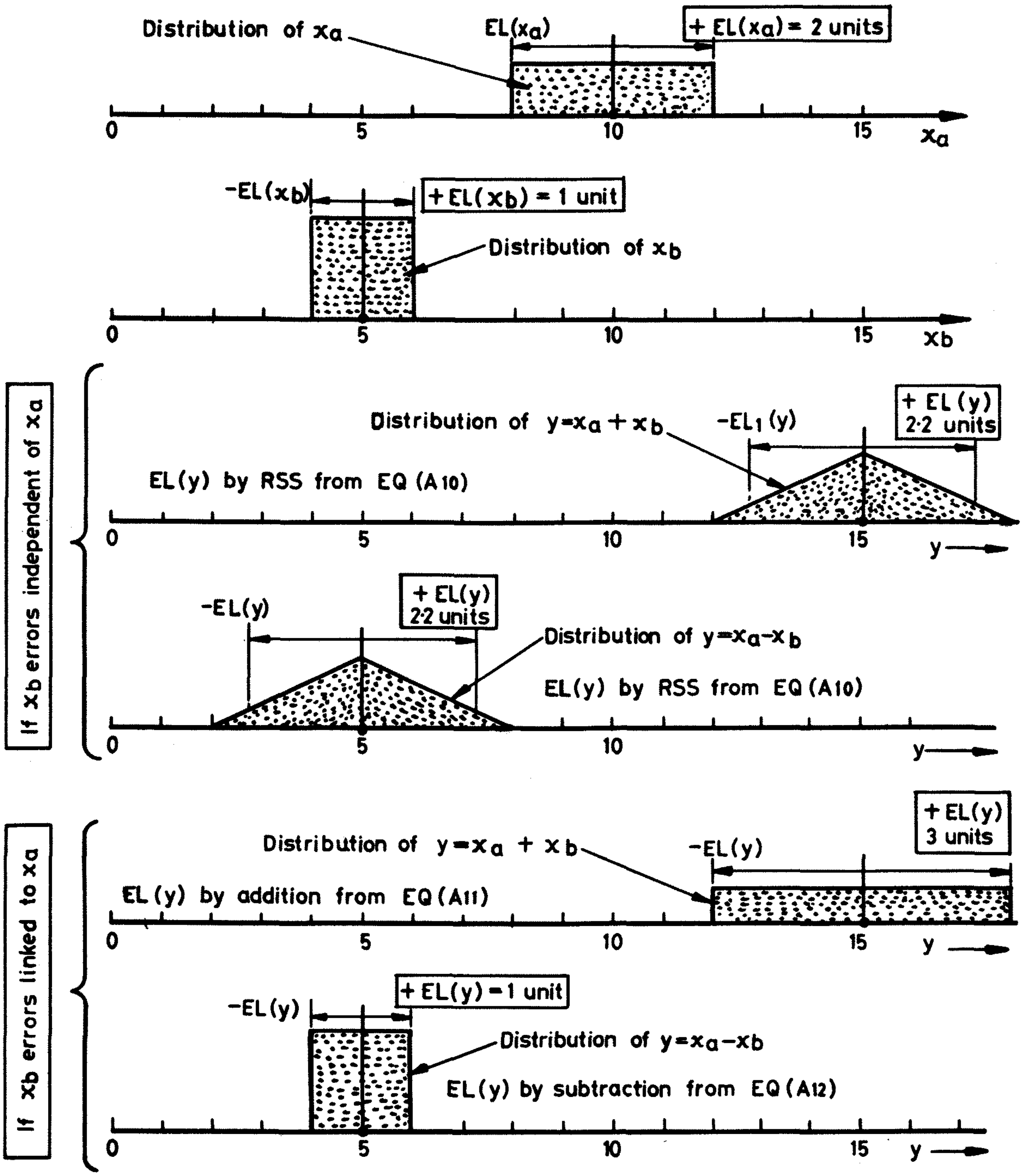

FIG. A3 COMBINATIONS OF INDEPENDENT AND NON INDEPENDENT ERRORS

TABLE I. ENGINE TEST CALIBRATION UNCERTAINTY

Airmeter calibration
Class I
Class II } ⟶ EL(mean C_{DA} curve) = 0.5% { This EL fossilised as Class III before transfer for use in Engine Tests }
Class III

Single-engine calibration uncertainty
Flight condition = 0.9 MN at low altitude, "High-Power", Dry

Input parameter x_i	$IC(C_G:x_i)$	Class I %EL(x_i)	Class I IC × EL	Class II %EL(x_i)	Class II IC × EL	Class III %EL(x_i)	Class III IC × EL
C_{DA}	0.2	——	——	——	——	0.5	0.10
P_{SA}	0.1	0.02	0.00	0.11	0.01	0.05	0.01
ΔP_A	0.1	0.10	0.01	0.37	0.04	0.21	0.02
T_{tA}	-0.1	0.09	-0.01	0.37	-0.04	0.40	-0.04
P_{CELL}	-0.5	0.04	-0.02	0.17	-0.08	0.08	-0.04
P_{t1}	1.4	0.01	0.01	0.10	0.14	0.05	0.07
T_{t1}	0.1	0.09	0.01	0.37	0.04	0.40	0.04
P_{S7}	-1.2	0.02	-0.02	0.07	-0.08	0.03	-0.04
A_8	-1.0	0.40	-0.40	0.97	-0.97	1.13	-1.13
F_L	0.3	0.23	0.07	0.30	0.09	0.61	0.18
W_{FCC}	-0.0	0.70	-0.00	0	0	0.05	-0.00
W_{FR}	0	0	0	0	0	0	0
CALVAL	0.0	——	——	——	——	0.15	0.00
%EL (Spot Point C_G) = $\sqrt{\Sigma(IC \times EL)^2}$		——	0.41	——	0.99	——	1.15
%EL (C_G curve)		——	$\frac{0.41}{\sqrt{4}} = 0.21$	——	$\frac{0.99}{\sqrt{4}} = 0.49$	——	1.15
Results of similar calculation for C_{D8}							
%EL (Spot Point C_{D8})		——	0.44	——	1.01	——	1.14
%EL (C_{D8} curve)		——	$\frac{0.44}{\sqrt{4}} = 0.22$	——	$\frac{1.01}{\sqrt{4}} = 0.50$	——	1.14
Results of similar calculation for C_X							
%EL (Spot Point C_X)		——	0.16	——	0.25	——	0.26
%EL (C_X curve)		——	$\frac{0.16}{\sqrt{4}} = 0.08$	——	$\frac{0.25}{\sqrt{4}} = 0.13$	——	0.26

GO TO TABLE II

TABLE II. UNCERTAINTY TRANSFER OF LINKED CALIBRATION COEFFICIENTS FROM ATF TO FLIGHT (ONE ENGINE OF TWIN-ENGINED AIRCRAFT)

Flight condition: 0.9 MN at low altitude, "High-Power", Dry

Case	From Table I %EL (separate coefficients)	Equation (37) $(A^2 + AB)[\%EL\ (C_G)]^2 + (B^2 + AB)[\%EL\ (C_{D8})]^2 - AB[\%EL\ (C_X)]^2$	%EL(F_N)
(a) Both engines calibrated in ATF: (a1) Independent of other engine (Classes I and II)	%EL (C_G) = $\sqrt{0.21^2 + 0.49^2}$ = 0.54 %EL (C_{D8}) = $\sqrt{0.22^2 + 0.50^2}$ = 0.55 %EL (C_X) = $\sqrt{0.08^2 + 0.13^2}$ = 0.15	$(4.75 - 3.34)0.54^2$ = 0.41 $+ (2.34 - 3.34)0.55^2$ = -0.30 $+ 3.34 \times 0.15^2$ = 0.08 0.19	$\sqrt{0.19}$ = 0.44%
(a) Both engines calibrated in ATF: (a2) Common to both engines (Class III)	%EL (C_G) = 1.15 %EL (C_{D8}) = 1.14 %EL (C_X) = 0.26	$(4.75 - 3.34)1.15^2$ = 1.87 $+ (2.34 - 3.34)1.14^2$ = -1.31 $+ 3.34 \times 0.26^2$ = 0.23 0.77	$\sqrt{0.77}$ = 0.88%
(b) Only one engine calibrated in ATF ∴ Classes I, II and III independent of other engine	%EL (C_G) = $\sqrt{0.21^2 + 0.49^2 + 1.15^2}$ = 1.27 %EL (C_{D8}) = $\sqrt{0.22^2 + 0.50^2 + 1.14^2}$ = 1.26 %EL (C_X) = $\sqrt{0.08^2 + 0.13^2 + 0.26^2}$ = 0.30	$(4.75 - 3.34)1.27^2$ = 2.27 $+ (2.34 - 3.34)1.26^2$ = -1.61 $+ 3.34 \times 0.30^2$ = 0.30 0.96	$\sqrt{0.96}$ = 0.98%
(c) Both engines calibrated in ATF "Old Theory"	%EL (C_G) = 1.27 %EL (C_{D8}) = 1.26	"Old Theory" 4.75×1.27^2 = 7.77 2.34×1.26^2 = 3.71 11.37	$\sqrt{11.37}$ = 3.38%

These linked calibration coefficient uncertainties become "fossilised" into Class III upon transfer from ATF to flight

GO TO TABLE III

TABLE III COMPLETE PREDICTION OF IN-FLIGHT THRUST UNCERTAINTY FOR TWIN ENGINE AIRCRAFT

Flight condition: 0.9 MN at low altitude, "high-power", dry

Part 1 Single engine $C_T = \dfrac{F_N}{qS}$

Input parameter x_i		IC $(C_T : x_i)$	Class I %EL	Class I IC × EL Independent	Class I IC × EL Common	Class II %EL	Class II IC × EL Independent	Class II IC × EL Common	Class III %EL	Class III IC × EL Independent	Class III IC × EL Common		FROM TABLE II – Fossilised errors
Linked calibration curves of C_G, C_{Ds}, C_X transferred from Engine Test Bed (see Table II)										0.44	0.88	← a	If both engines calibrated in ATF
										0.98	-	← b	If only this engine calibrated in ATF
										3 × 0.98	-	← c	If only this engine calibrated in SLSTB
										3 × 0.44	3 × 0.88	← d	If both engines calibrated in SLSTB
										3.38	-	← e	Both engines calibrated in ATF (Old Theory)
Aircraft instrumentation	P_{so}	-0.33	0.5		-0.16	0.5		-0.16	0.1		-0.03		
	T_{t1}	-0.18	0.5		-0.09	1.0		-0.18	1.0		-0.18		
	$(P_{to} - P_{so})$	-0.95	0.5		-0.47	0.5		-0.47	0.1		-0.10		
Engine instrumentation	$P_{s7} - P_{to}$	0.52	0.5	0.26		0.5	$0.26/\sqrt{2}$	$0.26/\sqrt{2}$	0.25		0.13		
	P_{t8}	0	-	-		-	-	-	-		-		
	P_{t18}	0	-	-		-	-	-	-		-		
	A_8	0.65	0.4	0.26		1.0	$0.65/\sqrt{2}$	$0.65/\sqrt{2}$	1.0		0.65		
	W_{FCC}	0.35	1.0	0.35		1.0	$0.35/\sqrt{2}$	$0.35/\sqrt{2}$	0		0		
	W_{FRH}	-	-	-		-	-	-	-		-		
	LCV	0.35	0	0		0	0	0	0.5		0.18		
	T_F	-0.13	0.5	-0.06		0.5	$-0.06/\sqrt{2}$	$-0.06/\sqrt{2}$	0.5		-0.06		
RSS separate classes $= \sqrt{\sum_i [IC \times EL]^2}$ = %EL(1 engine spot point C_T)		a		0.51	0.50		0.56	0.77		0.44	1.14	a →	
		b		0.51	0.50		0.56	0.77		0.98	0.72	b →	
		c		0.51	0.50		0.56	0.77		2.94	0.72	c →	
		d		0.51	0.50		0.56	0.77		1.32	2.74	d →	
		e		0.51	0.50		0.56	0.77		3.38	0.72	e →	

RSS all classes $\sqrt{\sum [IC \times EL]^2}$ = %EL(Single engine spot point C_T)

	Independent of other engine	Common to both engines	Combined	
a →	0.87	1.46	1.70	ATF
b →	1.24	1.17	1.70	ATF
c →	3.04	1.17	3.26	SLSTB
d →	1.52	2.89	3.26	SLSTB
e →	3.46	1.17	3.65	ATF

Part 2: Twin engined aircraft $C_T = \dfrac{F_{N1} + F_{N2}}{qS}$ — %EL (Twin engine spot point C_T)

2 engines calibration in ATF	a + a	⟶	$\left\{[0.87/\sqrt{2}]^2 + 1.46^2\right\}^{1/2}$	= 1.58%
1 engine calibration in ATF + 1 engine in SLSTB	b + c	⟶	$\left\{[1.24/2]^2 + [3.04/2]^2 + 1.17^2\right\}^{1/2}$	= 2.02%
2 engines calibration in SLSTB	d + d	⟶	$\left\{[1.52/\sqrt{2}]^2 + 2.89^2\right\}^{1/2}$	= 3.08%
Each engine calibration in different ATF	b + b	⟶	$\left\{[1.24/\sqrt{2}]^2 + 1.17^2\right\}^{1/2}$	= 1.46%
Each engine calibration in different SLSTB	c + c	⟶	$\left\{[3.04/\sqrt{2}]^2 + 1.17^2\right\}^{1/2}$	= 2.45%
2 engines calibration in ATF (Old Theory)	e + e	⟶	$\left\{[3.46/\sqrt{2}]^2 + 1.17^2\right\}^{1/2}$	= 2.71%